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10MAT2

Second Semester B.E. Degree Examination, December 2011
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose your answers for the following : **(04 Marks)**

i) The general solution of the equation $yp^2 + (x-y)p - x = 0$ is

A) $(x-y-c)(x^2+y^2-c)=0$ B) $(y-x-c)(x^2-y^2-c)=0$

C) $(y-x-c)(y^2-x^2-c)=0$ D) $(y-x-c)(x^2+y^2-c)=0$

ii) The given differential equation is solvable for x, if it is possible to express x in terms of,

A) x and y B) x and p C) y and p D) None of these

iii) The singular solution of the equation $y = px + \frac{a}{p}$ is

A) $y^2 = 4ax$ B) $x^2 = 4ay$ C) $x^2 = y$ D) $y^2 = x$

iv) The general solution of Clairaut's equation is,

A) $y = cx + f(c)$ B) $x = cy + f(c)$ C) $y = cx - f(c)$ D) None of these

b. Solve : p(p+y) = x(x+y). **(04 Marks)**

c. Obtain the general solution and the singular solution of the equation, $y = 2px + p^2y$. **(06 Marks)**

d. Obtain the general and singular solution of Clairaut's equation, $xp^3 - yp^2 + 1 = 0$. **(06 Marks)**

2 a. Choose your answers for the following : **(04 Marks)**

i) The particular integral of $(D^2 + a^2)y = \sin ax$ is

A) $-\frac{x}{2a}\cos ax$ B) $\frac{x}{2a}\cos ax$ C) $-\frac{ax}{2}\cos ax$ D) $\frac{ax}{2}\cos ax$

ii) The solution of the differential equation $y'' + y = 0$ satisfying the conditions y(0) = 1 and $y\left(\frac{\pi}{2}\right) = 2$ is

A) $y = \cos x - 2\sin x$ B) $y = 2\sin x - \cos x$

C) $y = \cos x + 2\sin x$ D) $y = C_1\cos x + C_2\sin x$

iii) P.I of $(D+1)^2y = xe^{-x}$ is,

A) $\frac{x}{6}e^{-x}$ B) $\frac{x^3}{6}e^{-x}$ C) $-\frac{x^3}{6}e^{-x}$ D) $\frac{x^2}{2}e^{-x}$

iv) P.I of $(D^2 + D)y = x^2 + 2x + 4$ is

A) $\frac{x^2}{3} + 4x$ B) $\frac{x^3}{3} + 4$ C) $\frac{x^3}{3} + 4x$ D) $\frac{x^3}{3} + 4x^2$

b. Solve : $(D-2)^2y = 8(e^{2x} + \sin 2x)$ **(04 Marks)**

c. Solve : $y'' - 2y' + y = x\cos x$ **(06 Marks)**

d. Solve : $\frac{dx}{dt} - 7x + y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$. **(06 Marks)**

3 a. Choose your answers for the following : **(04 Marks)**

i) The complementary function of the equation $x^2y'' - xy' + y = \log x$ is

A) $y = (C_1 + C_2x)e^x$ B) $y = (C_1 + C_2 \log x)x$
C) $y = (C_1 + C_2x)x$ D) $y = C_1e^x + C_2e^{-x}$

ii) The homogeneous linear differential equation whose auxillary equation has roots 1, -1 is

A) $x^2y_2 - xy_1 + y = 0$ B) $x^2y_2 - xy_1 - y = 0$
C) $y'' - y = 0$ D) $x^2y_2 + xy_1 - y = 0$

iii) To transform $xy'' + y' = \dfrac{1}{x}$ into a linear differential equation with constant coefficients put x = ...

A) e^t B) e^{-t} C) logt D) None of these

iv) The solution of $x^2y'' + xy' = 0$ is

A) $y = C_1 \cos x + C_2 \sin x$ B) $y = C_1e^x + C_2e^{-x}$
C) $y = a \log x + b$ D) $y = C_1 + 6x^3$

b. Solve $y'' - 6y' + 9y = \dfrac{e^{3x}}{x^2}$ by the method of variation of parameters. **(04 Marks)**

c. Solve : $(1 + x^2)\dfrac{d^2y}{dx^2} + (1 + x)\dfrac{dy}{dx} + y = 2\sin[\log(1 + x)]$. **(06 Marks)**

d. Solve by Frobenius method the equation: $4x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$. **(06 Marks)**

4 a. Choose your answers for the following : **(04 Marks)**

i) The solution of $\dfrac{\partial^2 z}{\partial y^2} = \text{Sin}(xy)$ is

A) $z = -x^2 \text{ Sin }(xy) + yf(x) + g(x)$ B) $z = -x^2 \text{ Cos }(xy) - yf(x) + g(x)$
C) $z = -\dfrac{\text{Sin}(xy)}{x^2} + yf(x) + g(x)$ D) None of these

ii) A solution of $(y - z)\, p + (z - x)\, q = x - y$ is

A) $x^2 + y^2 + z^2 = f(x - y - z)$ B) $x^2 + y^2 + z^2 = f(x + y + z)$
C) $x^2 - y^2 - z^2 = f(x + y + z)$ D) $x^2 + y^2 - z^2 = f(x + y + z)$

iii) The partial differential equation obtained from $z = ax + by + ab$ is

A) $px + qy + z = 0$ B) $px + qy + z^2 = 0$
C) $px - qy = z$ D) $px + qy = z$

iv) The partial differential equation obtained from $z = e^y f(x + y)$ is

A) $p + z = q$ B) $p - z = q$ C) $p - q = z$ D) None of these

b. Form the partial differential equation by eliminating the arbitrary functions from $z = f(y - 2x) + g(2y - x)$. **(04 Marks)**

c. Solve : $(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - xy$. **(06 Marks)**

d. Solve : $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$ by the method of separation of variables, given $u(0, y) = 2e^{5y}$. **(06 Marks)**

PART – B

5 a. Choose your answers for the following : **(04 Marks)**

i) $\displaystyle\int_0^2\int_0^x (x + y)\, dx\, dy = ...$

A) 0 B) 1 C) 3 D) 4

ii) $\int_0^\infty e^{-x^2} dx = \ldots\ldots$

A) $\sqrt{\pi}$ B) $\frac{\sqrt{\pi}}{2}$ C) $\sqrt{\frac{\pi}{2}}$ D) $\frac{\pi}{2}$

iii) The value of $\beta(2, 1) + \beta(1, 2)$ is

A) 0 B) $\frac{1}{2}$ C) 2 D) 1

iv) $\int_0^2 \int_1^3 \int_1^2 xy^2 z \; dz\, dy\, dx = \ldots\ldots$

A) 26 B) 25 C) 1 D) 0

b. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ and hence evaluate the same. **(04 Marks)**

c. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing to polar coordinates. **(06 Marks)**

d. Show that $\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$. **(06 Marks)**

6 a. Choose your answers for the following : **(04 Marks)**

i) If $\vec{F} = x^2 i + xyj$ then $\int_C \vec{F}.d\vec{r}$ from (0, 0) to (1, 1) along the line y = x is

A) 0 B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) None of these

ii) The value of $\iint_S (yz\, dy\, dz + zx\, dz\, dx + xy\, dx\, dy)$ where s is the surface of unit sphere $x^2 + y^2 + z^2 = 1$ is

A) 0 B) 4π C) $\frac{4\pi}{3}$ D) 10π

iii) A necessary and sufficient condition that the line integral $\int_L F.\, dR$ for every closed curve C is

A) Curl F = 0 B) div F = 0 C) Curl F ≠ 0 D) div F ≠ 0

iv) If V is the volume bounded by a surface S and $\vec{F}$ is a continuously differentiable vector function then $\iiint_V \text{div}\, \vec{F}\, dv = \ldots\ldots$

A) 0 B) $\iint_S \vec{F} \times \hat{n}\, ds$ C) $\iint_S \vec{F}.\hat{n}\, ds$ D) None of these

b. Using Green's theorem evaluate $\int_C \left[(xy + y^2)dx + x^2 dy\right]$ where C is bounded by y = x and $y = x^2$. **(04 Marks)**

c. Verify Stroke's theorem for the vector $\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by x = 0, x = a, y = 0, y = b. **(06 Marks)**

d. Using divergence theorem evaluate $\int_S \vec{F}.ds$ where $\vec{F} = 4xi - 2y^2 j + z^2 k$ and S is the surface bounded by the region $x^2 + y^2 = 4$, z = 0, z = 3. **(06 Marks)**

7 a. Choose your answers for the following : **(04 Marks)**

i) If $L\{f(t)\} = f(s)$ then $L\{e^{-at} f(t)\}$ is

A) $f(s-a)$ B) $f(s+a)$ C) $f(s)$ D) None of these

ii) $L\left\{\frac{\text{Sin at}}{t}\right\} = \ldots\ldots.$

A) $\text{Cos}^{-1}\left(\frac{s}{a}\right)$ B) $\tan^{-1}\frac{s}{a}$ C) $\frac{\pi}{2} + \tan^{-1}\frac{s}{a}$ D) None of these

iii) $L\{u(t+2)\} = \ldots\ldots..$

A) $\frac{e^{-2s}}{s^2}$ B) e^{2s} C) $\frac{e^{2s}}{s}$ D) $\frac{e^{-2s}}{s}$

iv) $L\{s(t)\} = \ldots\ldots.$

A) 0 B) e^{-as} C) ∞ D) 1

b. Find the value of $\int_0^\infty t^3 e^{-t}\,\text{Sin}\,t\,dt$ using Laplace transforms. **(04 Marks)**

c. If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a-t, & a \le t \le 2a \end{cases}$, where $f(t+2a) = f(t)$, show that $L\{f(t)\} = \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$. **(06 Marks)**

d. Express $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$ interms of unit step function and hence find its Laplace transform. **(06 Marks)**

8 a. Choose your answers for the following : **(04 Marks)**

i) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when n is

A) zero B) –ve integer C) +ve integer D) –ve rational

ii) $L^{-1}\left\{\frac{s}{(s-1)^3}\right\} = \ldots\ldots$

A) $e^{-t}(t+t^2)$ B) $e^t\left(t+\frac{t^2}{2!}\right)$ C) $t\,e^t + t^2 e^t$ D) None of these

iii) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\} = \ldots\ldots$

A) 2 Sin t B) 2 Cos h t C) Sin h t D) 2 Sin h t

iv) $L^{-1}\left\{\frac{s}{(2s+3)^2}\right\} = \ldots\ldots.$

A) $-\frac{1}{8}(2-3t)e^{\frac{-3t}{2}}$ B) $\frac{1}{8}(2-3t)e^{\frac{-3t}{2}}$ C) $2e^{\frac{-3t}{2}} - 3te^{\frac{-3t}{2}}$ D) None of these

b. Find $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$. **(04 Marks)**

c. Using convolution theorem evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. **(06 Marks)**

d. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method. **(06 Marks)**

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